On Overlap Index

Overlap Index (OI) - a measure of activating regions' spatial overlap and symmetry

Overlap Index is calculated according to the following formula:

$$OI = \frac{0.5(INC_w + DEC_w) - sep}{0.5(INC_w + DEC_w) + sep} \tag{1}$$

where INC_w and DEC_w is the respective width of increment and decrement activating regions (AR) and *sep* is the separation between centers of these regions. The distribution of OI is bimodal, with a gap between simple and complex cells ($OI_{simple} \leq 0.3$ and $OI_{complex} \geq 0.5$).

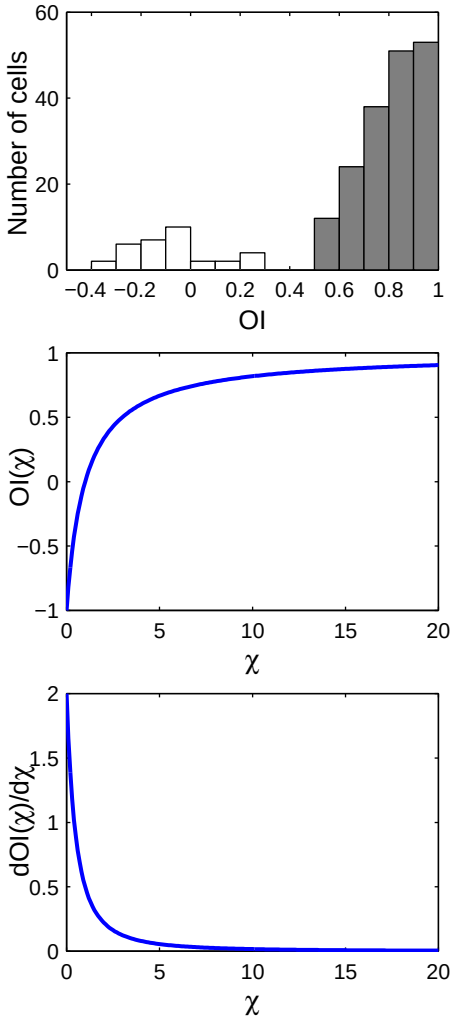

Figure 1: *Upper panel:* OI distribution. Open bars - simple cells, filled bars - complex cells (n=211). Bin width 0.1. *Middle panel:* Function $OI(\chi)$. *Lower panel:* Plot of derivative $OI'(\chi)$

A question may arise whether the bimodality of OI distribution can be an artefact of the method used for its calculation. If we denote $a = sep$, $b = 0.5(INC_w + DEC_w)$, and $\chi = b/a \geq 0$, then Eq. 1 can be written as a function of one variable χ (Fig. 1):

$$OI = \frac{b-a}{b+a} = \frac{\chi - 1}{\chi + 1} \tag{2}$$

The derivative $OI'(\chi)$ is:

$$\frac{dOI(\chi)}{d\chi} = (\chi+1)^{-1} - \frac{\chi-1}{(\chi+1)^2} \tag{3}$$

$OI'(\chi)$ is a nonlinear but *continuous monotonically decreasing* function. It shows that OI is very sensitive to small changes when χ is small, i.e. when *sep* is big (*simple cells*) or/and activating regions are small, and very robust when *sep* is small (*complex cells*) or/and activating regions are larger. However, since $OI'(\chi)$ does not have any maxima except at zero origin, the observed bimodality of OI can not be a result of some specific nonlinearity.

Other possible measures of separation/overlap

Since OI depends on the ratio of $b/a = \chi = 0.5(INC_w + DEC_w)/sep$ (i.e. the ratio of mean AR to separation between ARs), it is interesting to see if the distribution of χ is also bimodal. This distribution has a very long tail and does not seem to have a clear dip between simple and complex cells (Fig. 2, right panel), although there is a notch between simple and complex cells ($\chi_{simple} < 2$ and $\chi_{complex} > 3$) corresponding to the gap between simple and complex cells in the OI distribution.

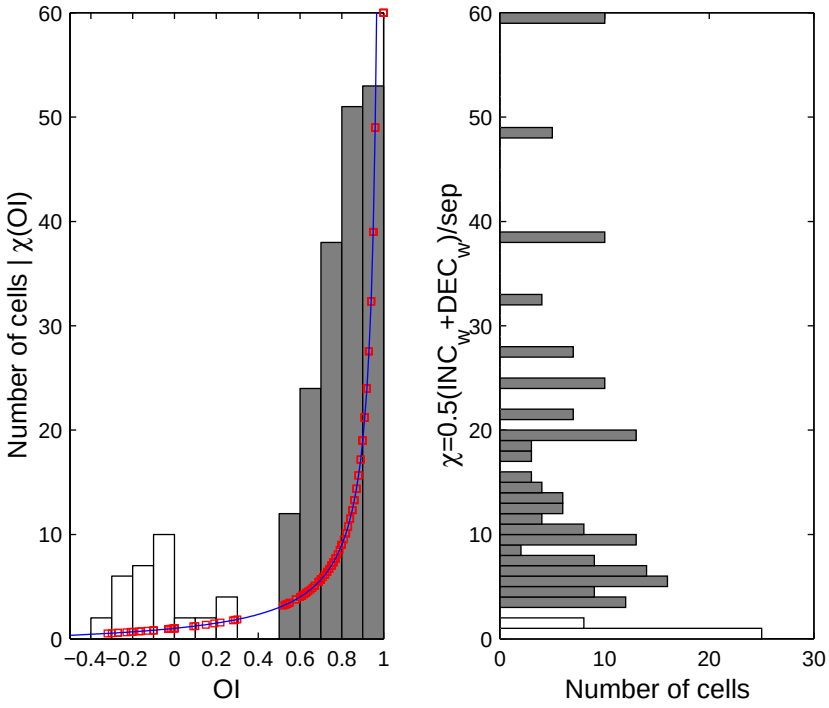

Figure 2: *Left panel:* OI distribution with $\chi(OI)$ function superimposed. Squares - data, solid line - "theoretical" curve. Note that OI was calculated with two digits precision, therefore the regular spacing with 0.01 OI shifts is seen in the data in the range $OI > 0.8$ even though most cells are concentrated in these two bins. Values of $\chi > 60$ are "truncated" to the $(1, 60)$ point. *Right panel:* Histogram of χ. Open bars - simple cells, filled bars - complex cells. Values of $\chi > 60$ are "truncated" to the last (59-60) bin. Bin width 1

Re-arranging Eq. 1 and substituting $\chi = 0.5(INC_w + DEC_w)/sep$, the χ can be expressed as function of OI (Fig. 2, left panel, red squares - data, solid line - "theoretical" curve):

$$sep = \frac{0.5(INC_w + DEC_w)\cdot(1-OI)}{1+OI} \tag{4}$$

$$\frac{0.5(INC_w + DEC_w)}{sep} = \chi(OI) = \frac{1+OI}{1-OI} \tag{5}$$

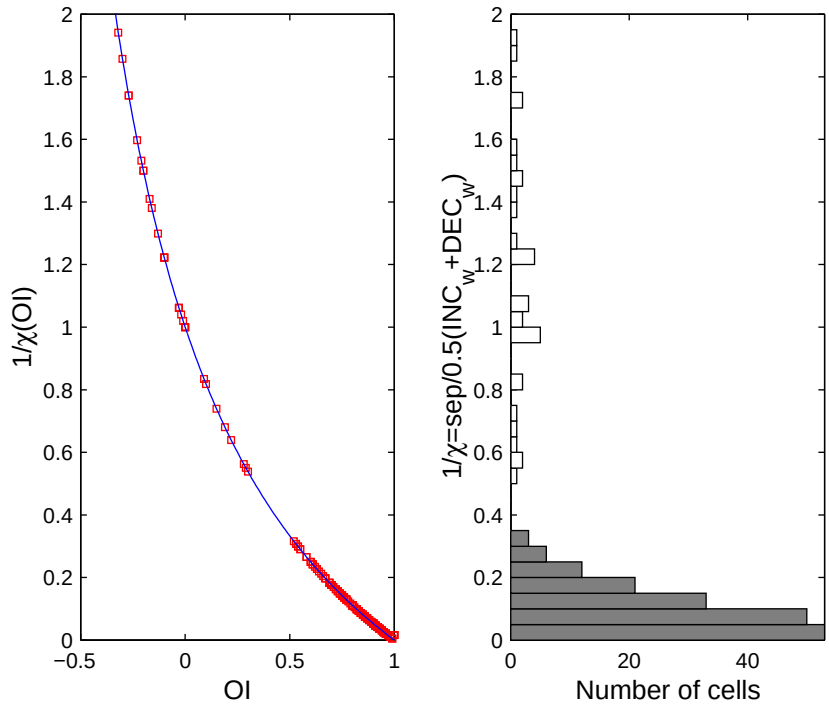

Figure 3: *Left panel:* $1/\chi(OI)$ function. Squares - data, solid line - "theoretical" curve. *Right panel:* Histogram of $1/\chi$. Open bars - simple cells, filled bars - complex cells. Bin width 0.05.

The distribution of χ is not dichotomous since the $\chi(OI)$ function changes at slow rate in the range of OI near zero (simple cells) but accelerates fast as OI approaches one (denominator $(1-OI) \rightarrow 0$, complex cells). Thus, all simple cells are concentrated in two lower bins while complex cells are widely dispersed, because the $\chi(OI)$ is very sensitive in the range $OI \geq 0.5$ (Fig. 2, right panel).

In fact, the inverse function $1/\chi$ seems to be a somewhat better than χ a "classifier", because of a smaller range and less "steepness" (Fig. 3). If we had a larger number of simple cells in our sample, we would probably see a more clear mode for simple cells around the expected ratio of one.

Also, the separation itself does not distinguish between simple and complex cells, because of variations in receptive field size (Fig. 4).

The last measure that we consider is the ratio of an increment and decrement ARs' overlap zone (OZ) to a classical RF width (CRF). This measure is identical to the Overlap Index except for those complex cells in which one AR is completely within the other, and thus the CRF size equals to the largest of two ARs:

$$OZ = INC_w + DEC_w - CRF \tag{6}$$

$$CRF = max\left\{0.5 \cdot INC_w + 0.5 \cdot DEC_w + sep, INC_w, DEC_w\right\} \tag{7}$$

In case when $CRF = 0.5 \cdot INC_w + 0.5 \cdot DEC_w + sep$

$$\frac{OZ}{CRF} = \frac{INC_w + DEC_w - (0.5 \cdot INC_w + 0.5 \cdot DEC_w + sep)}{0.5(INC_w + DEC_w) + sep} = \frac{0.5(INC_w + DEC_w) - sep}{0.5(INC_w + DEC_w) + sep} = OI \tag{8}$$

The OZ/CRF distribution is shown in (Fig. 5), and it can be seen that most cells are distributed along the equality diagonal.

In conclusion, the Overlap Index seems to be the best "classifier" to distinguish between "overlapping" (complex) and "non-overlapping" (simple) cells.

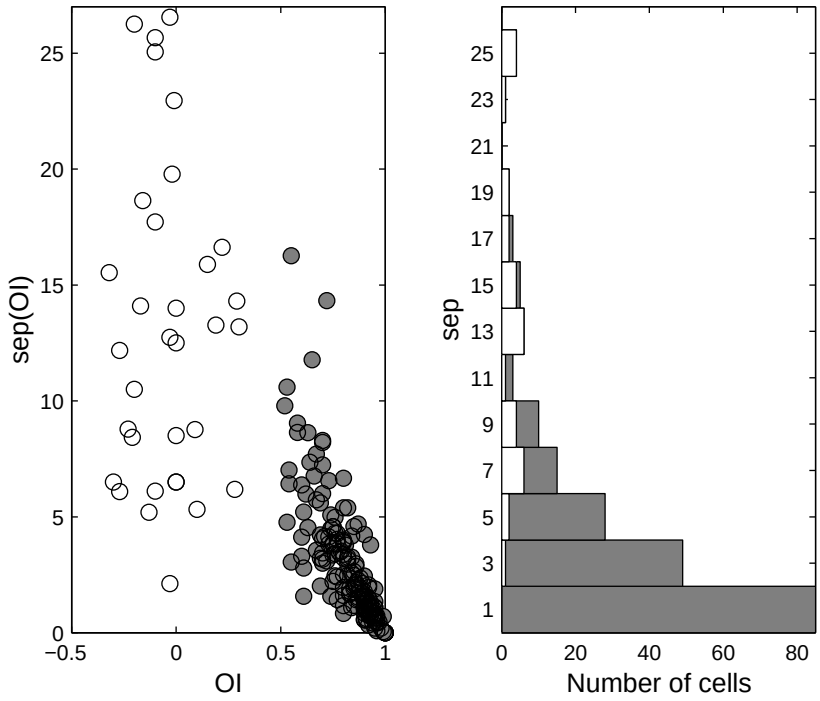

Figure 4: *Left panel:* Separation (sep, minarc) as function of OI. *Right panel:* Histogram of separation. Open bars - simple cells, filled bars - complex cells. Bin width 2 minarc.

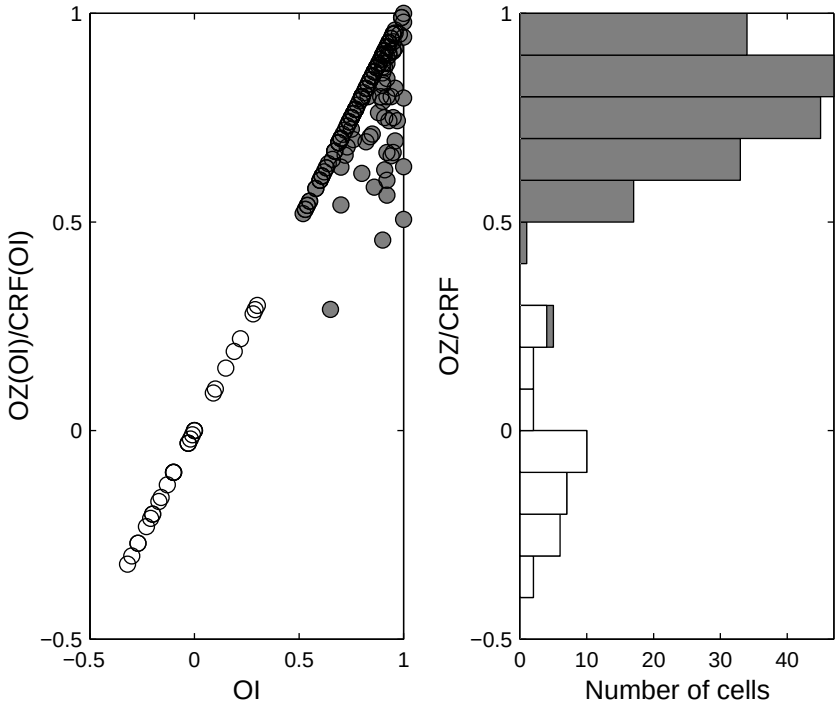

Figure 5: *Left panel:* Overlap zone (OZ, minarc) as function of OI. *Right panel:* Histogram of OZ. Open bars - simple cells, filled bars - complex cells. Bin width 0.1.